One Modulo Three Super Mean Labeling of Some Graphs

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**Abstract:** In this paper, we introduce a new labeling called one modulo three super mean labeling. A graph G is said to be **one modulo three super mean graph** if there is an injective function from the vertex set where q is the number of edges e = uv is defined as \* (e = uv) = if f(u)+f(v) is even if f(u)+f(v) is odd Then is called one modulo three super mean labeling if\*(e) = e . A graph admits a one modulo three super mean labeling is called one modulo three super mean graph. We proved that variety of graphs such as Path graph Pn, Comb graph, Sub division of Comb graph, the graphs Pn+Vk (1) and Pn ʘ S2 are identified as one modulo three Super mean graphs.

**Keywords:** Super mean labeling, one modulo three super mean labeling, connected graphs, comb graph, Sub division of Comb graph

# INTRODUCTION

Graph labeling plays an important role in graph theory. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. All graphs considered here are simple, finite and undirected. For standard terminology and notations we follow F. Harary [1].

S. Somasundaram and R. Ponraj introduced mean labeling of graphs [2]. Vaidya and et.al have investigated several new families of mean graphs [3]. Nagarajan and et.al have found some new results on mean graphs [4]. P.Jeyanthi and A. Maheswari introduced one modulo three mean labeling of graphs [5]. Some new mean graphs are discussed by Vaidhya et.al. [6]. The concept of super mean labeling was studied by D. Ramya et al. [7] [8] [9] [10].

The key objective of the current study is to investigate a number of graphs such as Path graph Pn, Comb graph, Sub division of Comb graph, the graphs Pn+Vk (1) and Pn ʘ S2 whether or not they admit one modulo three super mean labeling.

# PRELIMINARIES

DEFINITION: 2.1

A function is called a graceful labeling of a graph G if is injective and the induced function \* : defined as \*(e is bijective.

DEFINITION: 2.2

A graph G with p vertices and q edges is a mean graph if there is an injective function from the vertices of G to such that when each edge is called mean labeling with if is even and if is odd then the resulting edge are distinct.

DEFINITION:2.3

A graph G is said to be super mean graph if there is an injective function from the vertex set where q is the number of edges e = uv is defined as

\* (e = uv) = if f(u)+f(v) is even

if f(u)+f(v) is odd

Then is called super mean labeling if \*(e) = e . A graph admits a super mean labeling is called super mean graph.

DEFINITION: 2.4

A graph G is said to be one modulo three mean graph if there is an injective function from the vertex set of G to the set and either or where q is the number of edges of G and induces a bijection \* from the edge set of G to and given by \* (uv) = and the function is called one modulo three mean labeling of G [7].

DEFINITION:2.5

A graph G is said to be one modulo three super mean graph if there is an injective function from the vertex set where q is the number of edges e = uv is defined as

\* (e = uv) = if f(u)+f(v) is even

if f(u)+f(v) is odd

Then is called one modulo three super mean labeling if \*(e) = e . A graph admits a one modulo three super mean labeling is called one modulo three super mean graph.

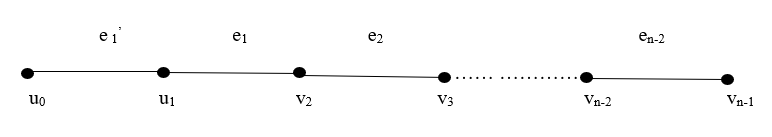
# Main Results

**Theorem 3.1:** The connected graph Pn is a one modulo three super mean graph.

**Proof:** Let be the vertices and be the edges.

We define the vertex labeling as

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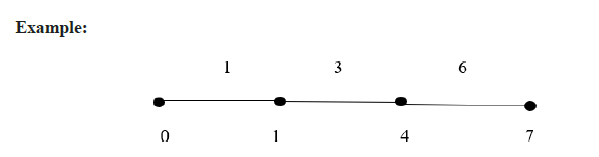
**Figure 1:** The path graph Pn satisfies the one modulo three super mean labeling.

Then the induced edge labeling for Pn is defined as

*)*

Here the edge labels are distinct. Hence *f* is a one modulo three super mean labeling of Pn. Hence the graph Pn is called a one modulo three super mean graph.

***Example:*** The path graph P4  is a one modulo three super mean labeling.

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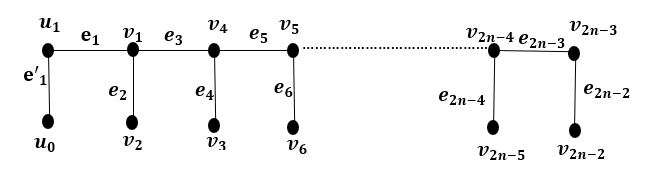
**Figure 2:** The path graph P4 a one modulo three super mean graph.

**Theorem 3.2:** The comb graph satisfies one modulo three super mean labeling.

**Proof:** Let be the vertices and be the edges.

We define the vertex labeling as

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**Figure 3:** The comb graph is a one modulo three super mean graph

We define edge labeling as

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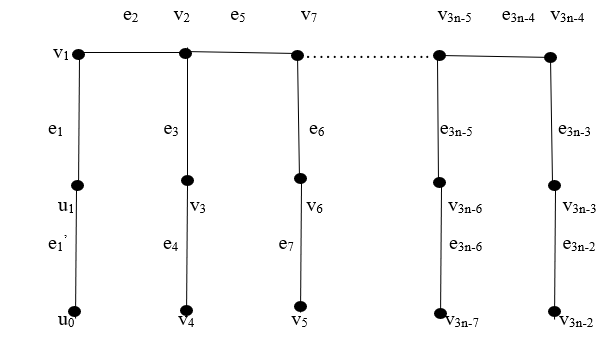
Here the edge labels are distinct. Hence f is a one modulo three super mean labeling of comb graph. Therefore the comb graph is a one modulo three super mean graph.

**Theorem 3.3:** The sub division of a comb graph satisfies one modulo three super mean labeling.

**Proof:** Let be the vertices and be the edges. Clearly it has (3n – 2) edges and vertices.

We define the vertex labeling as

f(ui) = i, i = 0 & 1 and .



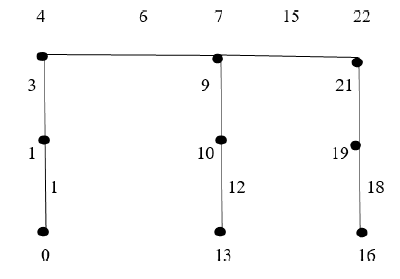
**Figure 4:** The sub division of comb graph is a one modulo three super mean graph

Hence the induced edge labeling is also defined as

*)*

Here the edge labels are distinct. Hence *f* is a one modulo three super mean labeling for sub division of a comb graph. Therefore the sub division of comb graph is a one modulo three super mean graph.

***Example:***

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**Figure 5:** The sub division of a comb graph (P3)

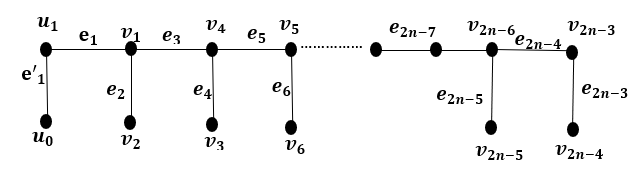
**Theorem 3.4:** The graph Pn+Vk (1) satisfies one modulo three super mean labeling.

**Proof:** Let be the vertices and be the edges.

This case is possible when n and k is even.

We define the vertex labeling as

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**Figure 6:** Pn+Vk (1) is a one modulo three super mean graph.

Then the induced edge labeling is defined as

*)*

Here the edge labels are distinct. Hence *f* is a one modulo three super mean labeling for the graph Pn+Vk (1). Therefore the graph Pn+Vk (1) is a one modulo three super mean graph.

***Theorem 3.5:*** *The graph* PnS2 *satisfies one modulo three super mean labelling only if n is even.*

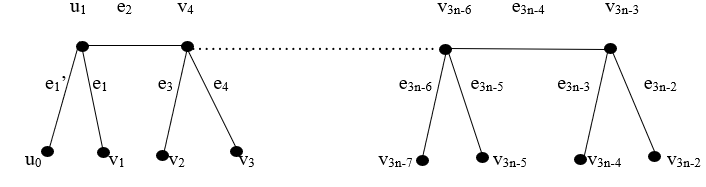
**Proof:** Let be the vertices and be the edges.

This case is possible when n is even.

The vertex labeling are defined as

*f(ui) = i; i = 0 & 1 and*

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**Figure 7:** The graph PnS2 is a one modulo three super mean labeling.

Then the induced edge labeling are defined as

*)*

and the edge labelings are distinct. Hence *f* is a one modulo three super mean labeling for the graph PnS2. Therefore the graph PnS2 is a one modulo three super mean graph.

# CONCLUSION

In the present study, we have verified that Connected graphs Pn, Comb graph, Sub division of Comb graph, Pn+Vk (1) and Pn ʘ S2 are one modulo three super mean labeling of graphs. It is imperative to note that this study is the first to demonstrate that the above-mentioned graphs admit one modulo three super mean labeling.

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